Introduction to Radio Astronomy (WBAS14001)

Final examination – Wednesday 31 January 2018 (09:00 – 12:00)

This examination contains 5 questions worth 20 points each. All questions should be answered. For full credit, all working and calculations should be shown, and full explanations given where required. Numerical answers should be in S.I. units, unless otherwise stated. Below is a list of constants and identities that may be used.

1. (a) Define (in words) what is meant by the *brightness temperature* of an object [2 points].

   (b) Show that, given the Planck function,

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}, \tag{1}$$

   where the symbols have their usual meaning, the flux density of an object in the low-frequency regime is given by,

$$S_\nu = \frac{2kT_b\Delta\Omega}{\lambda^2} \tag{2}$$

   where the symbols have their usual meaning [5 points].

   (c) Cygnus A is the most luminous radio galaxy in the low-frequency sky that can be observed with LOFAR. At 151 MHz, the source has a spectral luminosity of $L_{151\ \mathrm{MHz}} = 8.1 \times 10^{28}$ W Hz^{-1}. Given that Cygnus A is at a luminosity distance $D_L = 252.1$ Mpc, calculate the flux-density of the source at 151 MHz in Jansky's [4 points].

   (d) If the solid angle subtended by Cygnus A is $\Delta\Omega = 2$ arcmin^2, calculate the brightness temperature of the source at 151 MHz under the assumption that the emission is uniform over the source [3 points].

   (e) Given that at 74 MHz, Cygnus A has a spectral luminosity of $L_{74\ \mathrm{MHz}} = 13.1 \times 10^{28}$ W Hz^{-1} and that the broadband radio spectrum of Cygnus A can be described by a power-law, such that,

$$S_\nu \propto \nu^\alpha, \tag{3}$$

   calculate the spectral index, α, for the source between 74 and 151 MHz [2 points].

   (f) Based on your answer for part (e), explain whether this is consistent with a thermal or non-thermal radio source, and describe what the likely dominant mechanism is for the production of radio emission from Cygnus A at low radio frequencies [4 points].

2. (a) A distant radio-loud quasar, 4 arcsec in angular size, has a total un-resolved flux-density at 1.4 GHz of 600 mJy, which produces an antenna temperature of $T_A = 1$ K. Given that the antenna temperature is defined as,

$$T_\mathrm{A} = \frac{A_e}{2k}\int I_\nu(\theta,\phi)P_\mathrm{n}(\theta,\phi)d\Omega, \tag{4}$$

   where the symbols have their usual meaning, calculate the diameter of this antenna if it where an ideal circular paraboloidal surface [3 points].

   (b) Confirm that the quasar can be considered as an un-resolved point-source for this observation, under the assumption that the antenna has a uniform illumination pattern [3 points].

(c) The paraboloidal surface used for this observation is not ideal, but has surface errors $\sigma = 8$ mm, as defined by Ruze's equation,

$$\eta_s = \exp\left[-\left(\frac{4\pi\sigma}{\lambda}\right)^2\right]. \tag{5}$$

Assuming that these surface errors dominate the aperture efficiency, calculate what the antenna temperature would be for the same observation [4 points].

(d) Describe (in words) what other factors can contribute to the aperture efficiency of an antenna, in particular discuss where they come from and how they can be minimised [6 points].

(e) Describe what is meant by an equatorial and alt-azimuth mount for a radio antenna, and give at least one advantage that each has over the other for making astronomical observations [4 points].

3. (a) Define (in words) what is meant by the *system temperature* of a receiver system [2 points].

(b) Write down an equation that expresses the system temperature in terms of the 5 dominant noise sources, and explain what each of these noise sources are [6 points].

(c) The point-source sensitivity of a parabola dish telescope is defined by the radiometer equation,

$$\sigma_{\rm rms} = \frac{2\,k\,T_{\rm sys}}{A_{\rm eff}\sqrt{\Delta\nu\,t_{\rm int}}}, \tag{6}$$

where the symbols have their usual meaning. Calculate the system equivalent flux density, in Jansky's, for the Lovell telescope at Jodrell Bank, assuming a diameter of 76 m, an aperture efficiency of 85% and a system temperature of 45 K [4 points].

(d) Calculate the point-source sensitivity, in Jansky's, for an observation with a bandwidth of 512 MHz and an integration time of 10 mins (you may neglect the effect of source confusion) [2 points].

(e) In reality there will be gain variations within the receiver system over time. Show that these gain variations are equivalent to, and therefore can not be distinguished from, variations in the system temperature [4 points].

(f) Describe an observational method that can be used to measure the antenna temperature due to the target source, which removes these variations in the system gain and temperature over time for a single-dish telescope system [2 points].

4. (a) A medium that is made up of free electrons is considered ionised, with a corresponding refractive index that is given by,

$$n^2 = 1 - \left(\frac{\nu_{\rm p}}{\nu}\right)^2 \tag{7}$$

where the symbols have their usual meaning. Describe (in words) three effects an ionised medium can have on the propagation of continuum radio waves, and give examples of the typical refractive index and observing frequency in each case [9 points].

(b) A pulsar emits radiation within our galaxy that passes through an ionised medium, resulting in a *dispersion* of the radio waves. Draw a schematic dynamic spectrum (frequency as a function of time) showing this dispersive effect [2 points].

(c) From an observation with the Green Bank Telescope of a pulsar at a distance of 1 kpc, the dispersion measure is found to be 10 pc cm^{-3}. Calculate the average electron density along the line-of-sight to this pulsar [2 points].

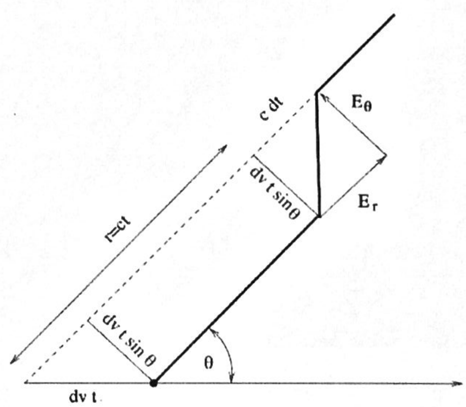
(d) Given that the plasma frequency is given by,

$$\omega_{\rm p}^2 = \frac{N_e\, e^2}{\epsilon_0\, m_e} \tag{8}$$

where the symbols have their usual meaning, calculate the refractive index of the ionised medium along the line-of-sight to the pulsar at 150 MHz [3 points].

(e) A recent *fast radio burst* was detected with a dispersion measure of $\sim$ 1000 pc cm^{-3}. Write a brief discussion as to how such a large dispersion measure could be found for an object [4 points].

5. (a) Show that the total power emitted by an accelerated charge, with electric field lines as shown in bold below,



is given by Larmor's formula (in cgs units),

$$P = \frac{2}{3}\frac{q^2 a^2}{c^3}, \tag{9}$$

where the symbols have their usual meaning [10 points].

(b) Draw a schematic diagram for the spectral energy distribution of a typical star-forming galaxy between 1 GHz and 1000 GHz, showing clearly the main emission mechanisms and their dependance on frequency [3 points].

(c) Describe how each of these mechanisms results in the emission of electro-magnetic radiation at radio wavelengths [3 points].

(d) The star-bursting galaxy Arp 220 is at a distance of 77 Mpc and has a 1.4 GHz flux-density of $S_{1.4\ \rm GHz}$ = 302 mJy. Using the radio–far-infrared correlation,

$$q_{\rm IR} = \log_{10}\left(\frac{L_{\rm IR}}{3.75\times10^{12}\ {\rm Hz} * L_{1.4\ \rm GHz}}\right) \tag{10}$$

where the symbols have their usual meaning, and the parameter $q_{\rm IR}$ = 2.4, calculate the expected far-infrared luminosity of Arp 220 [3 points].

(e) Hence, using the Kennicutt relation,

$$\frac{\text{SFR}}{(\text{M}_\odot\ \text{yr}^{-1})} = 1.71 \times 10^{-10} \frac{L_{\text{IR}}}{(\text{L}_\odot)}, \tag{11}$$

calculate the star-formation rate (SFR) of Arp 220 in solar-masses per year ($\text{M}_\odot\ \text{yr}^{-1}$) [1 point].

Useful constants

Boltzmann constant $k = 1.3806488 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
Speed of light in a vacuum $c = 1/\sqrt{\mu_0 \epsilon_0} = 299\,792\,458$ m s^{-1}
Permittivity of free-space $\epsilon_0 = 8.85418782 \times 10^{-12}$ A^2 s^4 kg^{-1} m^{-3}
1 $\text{L}_\odot \equiv 3.846 \times 10^{26}$ W
1 Mpc $\equiv 3.08568 \times 10^{22}$ m
1 Jansky (Jy) $\equiv 1 \times 10^{-26}$ W m^{-2} Hz^{-1}
Mass of the electron $m_e = 9.11 \times 10^{-31}$ kg
Electric charge on the electron $e = 1.60217662 \times 10^{-19}$ C

Useful identities

The Taylor expansion: $e^x \approx 1 + x/1! + x^2/2! + x^3/3! + ...$
$\int_0^\pi \sin^3\theta d\theta = 4/3$